CAD-Oriented Equivalent Circuit Modeling of Step Discontinuities in Rectangular Waveguides

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***Abstract*— A novel equivalent circuit model for step discontinuities in rectangular waveguides is presented. The equivalent circuit model is based on a modal analysis and enables full-wave characterization of the step-discontinuity by circuit analysis on commercial simulators. The equivalent circuit model of the H-plane step has been implemented on Libra to demonstrate its accuracy. Results of the circuit analysis were found to be in virtually perfect agreement with the conventional mode-matching solution based on incident and reflected wave amplitudes.**

I. Introduction

THE analysis of step discontinuities in rectangular waveguides has received considerable attention over the years. Of the many full-wave analysis techniques available, modal analysis methods [1] are often preferred as they are computationally efficient and provide physical insight.

During the past few years, several full-wave modal analysis methods for single and cascaded step discontinuities have been presented. For example, a scattering matrix formulation has been given in [2], while an admittance matrix formulation has been described in [3]. In order to avoid the relative convergence phenomenon [4], a rigorous approach considering the correct edge behavior of the field has been introduced in [5].

Although several of the full-wave methods refer to multiport network and equivalent circuit modeling of the discontinuity, they do not lend themselves for implementation in commercial circuit simulators. In this letter, a novel equivalent circuit description of the step discontinuity is presented. The computer-aided design (CAD)-oriented equivalent circuit representation, which is based on a modal analysis, enables full-wave analysis of step discontinuities on commercial microwave circuit simulators. As a consequence, no specific computer code for the analysis of waveguide circuits containing step discontinuities is required.

II. The Step Discontinuity

The complete equivalent circuit representation of the full-wave step discontinuity problem can be derived directly from a modal analysis, as shown below. In this letter, only the H-plane step discontinuity is discussed. The derivation for the E-plane step is similar.

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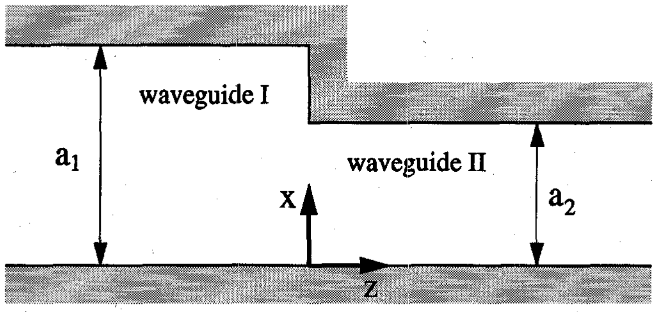


Fig. 1. Top view of an H-plane step discontinuity in a rectangular waveguide.

A. Modal Analysis

The H-plane step discontinuity is illustrated in Fig. 1. The transverse electric and magnetic fields in each waveguide can be expressed in terms of expansions in the transverse modal fields [6]. They are given in waveguide I by

$$E_y^{\rm I}(x,z) = \sum_{m=1}^{M} V_m^{\rm I}(z)\phi_m^{\rm I}(x)$$
$$H_x^{\rm I}(x,z) = -\sum_{m=1}^{M} I_m^{\rm I}(z)\phi_m^{\rm I}(x) \quad (1)$$

and in waveguide II by

$$E_y^{\rm II}(x,z) = \sum_{n=1}^{N} V_n^{\rm II}(z)\phi_n^{\rm II}(x)$$
$$H_x^{\rm II}(x,z) = -\sum_{n=1}^{N} I_n^{\rm II}(z)\phi_n^{\rm II}(x). \quad (2)$$

$V_k^{\rm I,II}(z)$ and $I_k^{\rm I,II}(z)$ are the total voltage and current amplitudes of waveguide mode k in region I and II, respectively. The expansions have been truncated to M modes in waveguide I and N modes in waveguide II. The number of expansion terms, M and N, is typically chosen such that M/N is as close as possible to the corresponding ratio in waveguide width, a_1/a_2, in order to avoid relative convergence problems [3], [4].

The continuity of the transverse fields at the discontinuity at $z = 0$ yields for the electric field

$$E_y^{\rm I}(x,0) = \begin{cases} 0 & \text{on conducting boundary} \\ E_y^{\rm II}(x,0) & \text{on aperture} \end{cases} \quad (3)$$

and for the magnetic field

$$H_x^{\rm I}(x,0) = H_x^{\rm II}(x,0) \quad \text{on aperture.} \quad (4)$$

Following the mode-matching procedure as described in [3] and [7], two systems of linear equations relating the voltage and current amplitude coefficients in each waveguide are found as

$$V_m^{\mathrm{I}} = \sum_{n=1}^{N} C_{mn} V_n^{\mathrm{II}} \tag{5}$$

and

$$I_n^{\mathrm{II}} = \sum_{m=1}^{M} C_{mn} I_m^{\mathrm{I}}. \tag{6}$$

The coefficients C_{mn} are the mode-coupling (overlap) integrals given by

$$C_{mn} = \int_0^{a_2} \phi_m^{\mathrm{I}}(x) \phi_n^{\mathrm{II}}(x)\, dx. \tag{7}$$

Equations (5) and (6) may be expressed compactly in matrix form as

$$\begin{aligned} \mathbf{V}^{\mathrm{I}} &= \mathbf{C}\mathbf{V}^{\mathrm{II}} \\ \mathbf{I}^{\mathrm{II}} &= \mathbf{C}^T \mathbf{I}^{\mathrm{I}} \end{aligned} \tag{8}$$

with the superscript T denoting transposition [3], [7].

It is noted that the conventional modal description of step discontinuities in terms of incident and reflected waves does not directly render CAD-oriented equivalent circuit representations. In contrast, the formulation in terms of total voltage and current amplitudes [3], [6], [7] leads directly to equivalent circuit models that can be implemented on commercial circuit simulators, as described below.

B. Equivalent Circuit Model

Equations (5) and (6) lead to a simple network description of the step discontinuity: voltages in region I are controlled by voltages in region II and currents in region II are controlled by currents in region I. The corresponding equivalent circuit representation of the step discontinuity is shown in Fig. 2. It consists of linear voltage-controlled voltage sources and current-controlled current sources, which is similar to the network description of coupled transmission lines [8]. Higher-order modes are represented here by their modal impedances

$$Z_k^{\mathrm{I,II}} = \frac{j\omega\mu}{\sqrt{(k\pi/a_{1,2})^2 - \omega^2\mu\epsilon}}\,. \tag{9}$$

For cascaded steps, the dependent sources are connected to dispersive transmission lines that correspond to the fundamental and higher-order modes of the uniform waveguide section.

It is clear from this equivalent circuit representation that the step discontinuity only couples energy between the waveguide modes, but does not store energy. Energy storage is accomplished in the evanescent (reactive) fields excited at the discontinuity, represented in the equivalent circuit model in Fig. 2 by reactive modal impedances as terminations. Hence, the equivalent circuit model provides a full-wave description of the step discontinuity problem.

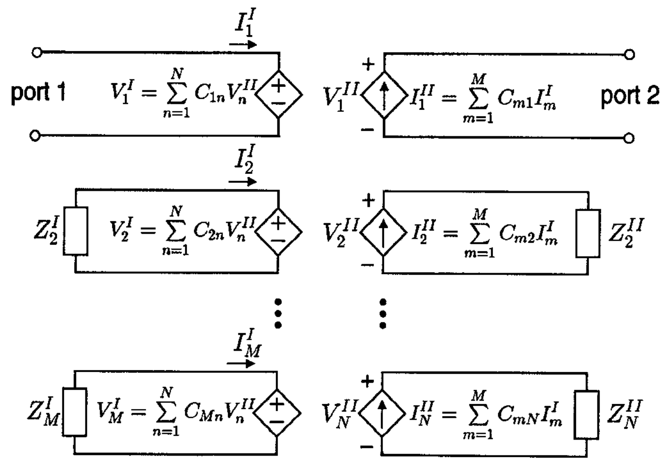


Fig. 2. Full-wave equivalent circuit of an H-plane step discontinuity. The quantities $Z_k^{\mathrm{I,II}}$ are the modal impedances of mode k in waveguides I and II, respectively. The gain coefficients C_{mn} of the ideal dependent sources are given by the overlap integrals in (7). M and N modes are retained in regions I and II, respectively.

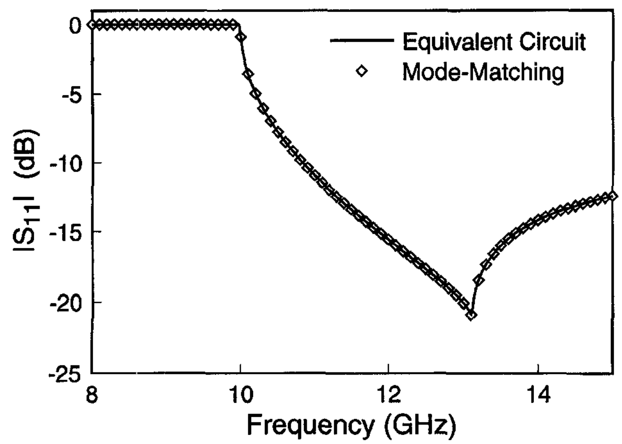
III. Results

The equivalent circuit model of the H-plane step discontinuity has been implemented on the commercial microwave simulator Libra [9]. The simulator provides element models for dependent sources and a dispersive transmission line model for the TE_{10} mode of a rectangular waveguide. Higher-order modes can be modeled by appropriate rescaling of the waveguide width. Since the available transmission line model for the TE_{10} mode uses the power-voltage definition of impedance, transformation to modal impedance has been achieved with ideal transformers. The overlap integrals in (7) giving the gain coefficients of the dependent sources are solved in closed form and computed by Libra.

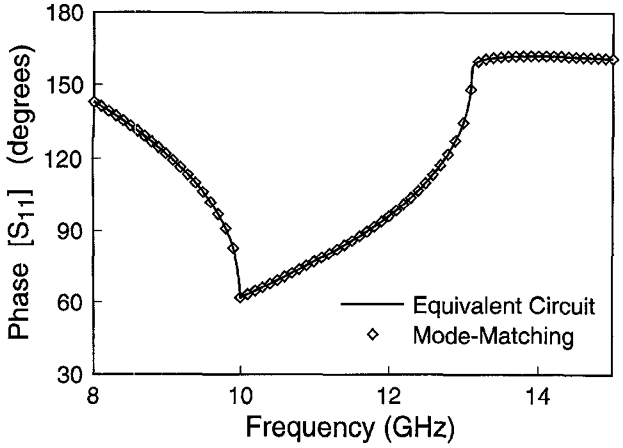
Fig. 3 shows the result of a representative Libra simulation of an H-plane step discontinuity in a WR-90 waveguide with width $a_1 = 22.86$ mm to width $a_2 = 15$ mm. Also included in the figure are computational results based on a mode-matching formulation in terms of incident and reflected wave amplitudes. In both cases, seven modes have been retained in the larger waveguide and five modes in the smaller waveguide in order to avoid relative convergence problems. Virtually perfect agreement between the Libra circuit simulation and the mode-matching calculation is apparent. It should be noted that waveguide II is cutoff below 10 GHz, whereas above 13.1 GHz two modes can propagate in waveguide I. The equivalent circuit representation also correctly computes the magnitude and phase of the scattering parameters in these frequency ranges.

IV. Conclusion

A novel equivalent circuit representation of the H-plane step discontinuity in a rectangular waveguide has been presented. The CAD-oriented equivalent circuit model enables full-wave analysis of the step discontinuity on commercial microwave circuit simulators. The equivalent circuit model of the H-plane step has been implemented on Libra to demonstrate



(a)



(b)

Fig. 3. (a) Magnitude and (b) phase of S_{11} as function of frequency of an H-plane step discontinuity with a_1 = 22.86 mm and a_2 = 15 mm.

its accuracy. Results of the circuit analysis were found to be in virtually perfect agreement with the conventional mode-matching solution based on incident and reflected wave amplitudes.

Since the step discontinuity is a basic building block, full-wave simulation of a variety of waveguide components including filters and matching networks can be performed entirely inside circuit simulators. The proposed equivalent circuit models and implementation in commercial CAD programs provide an alternative to other approaches that combine separately programmed full-wave methods with circuit CAD tools (e.g., [10]). The potential benefits of using commercial simulators in the analysis and design of waveguide components include: costumized codes for the design of waveguide components and circuits are not required and full advantage can be taken of the available design, optimization, and graphical user interface capabilities of the CAD tool. In addition, implementation of the full-wave analysis and design tool entirely inside a circuit simulator should offer other perfomance advantages over existing waveguide CAD tools.

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